ALGEBRA 2

5.1 END BEHAVIOR OF POLYNOMIALS

Warm up:

1. State the degree and leading coefficient of $f(x) = -8x^5 + 5x^2 - 15$.

2. State the degree and leading coefficient of $f(x) = 4 + 3x^2 + 6x^3$.

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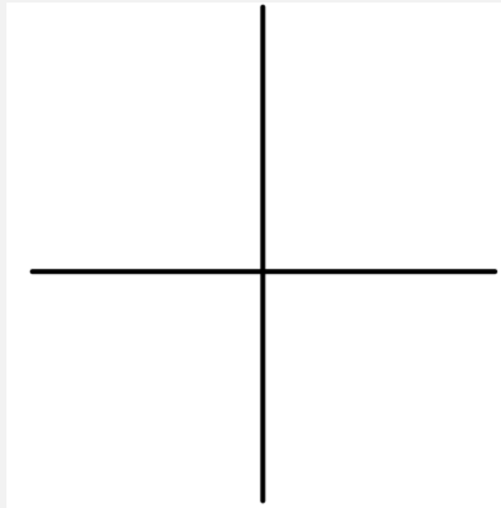
Degree	Leading Coefficient	Example of a Polynomial Function	Graph Shape

Big Idea Formal Notation for Describing End Behavior	

Example 1: Describe the end behavior of the graph using both words and formal notation.

a. $f(x) = x^3 - 2x^2 - 3x$

b. $y = -x^4 + 5x^2 - x + 1$

c. $g(x) = -x^5 - 5x$

d. $h(x) = x^4 - 7x^2 + x + 5$

Example 2: Starting with $f(x) = 2$, add a term to the polynomial at each step so that the graph has the specified shape.

a. up on the left and down on the right

b. up on both ends

c. down on both ends:

TRY IT: Try the problem below on your own. Then scroll down to check the answer. Ask Ms. McNabb if you need help.

Describe the end behavior of the graph using both words and formal notation.

$f(x) = -x^7 + 2$

ANSWER:

The graph points upward on the left and downward on the right end.

As $x \to -\infty, y \to \infty$ and as $x \to \infty, y \to -\infty$.

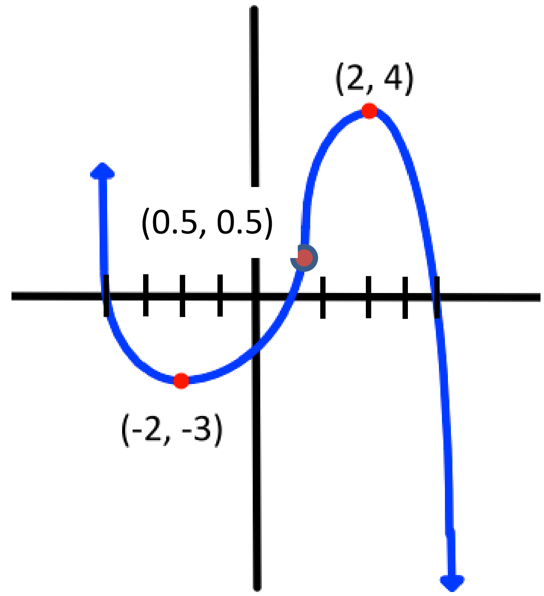
5.2 POLYNOMIAL GRAPH CONCEPTS

Warm up:

Describe the end behavior of $f(x) = -8x^5 + 5x^2 - 15$.

Big Idea **Polynomial Graphs**	**Turning points of polynomial functions:** - The y-coordinate of a turning point is a **local maximum** of the function if the point is higher than all nearby points. - The y-coordinate of a turning point is a **local minimum** of the function if the point is lower than all nearby points. - A function is **increasing** on an interval if as x increases, then y increases. (positive slope) - A function is **decreasing** on an interval if as x increases, then y decreases. (negative slope) - A polynomial function of degree n has AT MOST $n-1$ turning points.

Example 1: Complete the following statements about the polynomial whose graph is shown.



- The polynomial shown has turning points at:

- The function has a local minimum of:

- The function has a local maximum of:

- The function has roots of:

- The function is concave up on the interval:

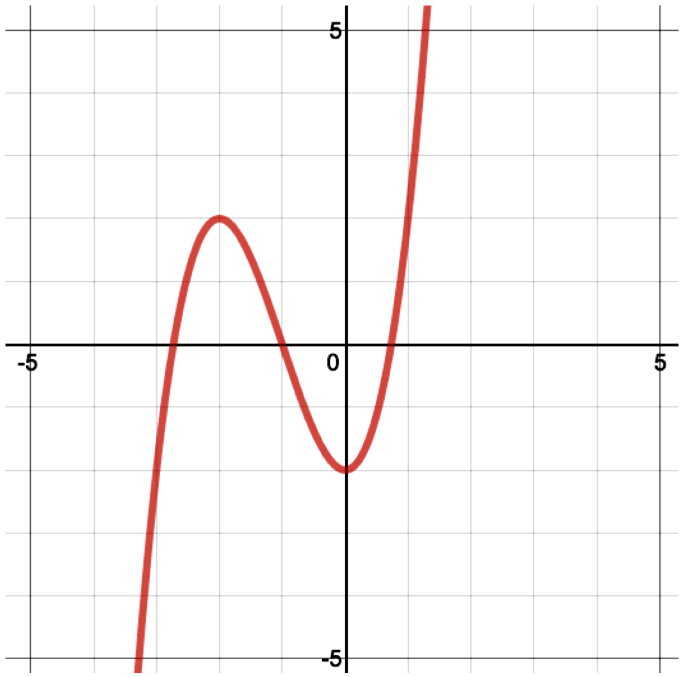
- The function is concave down on the interval:

- The function is increasing on the interval(s):

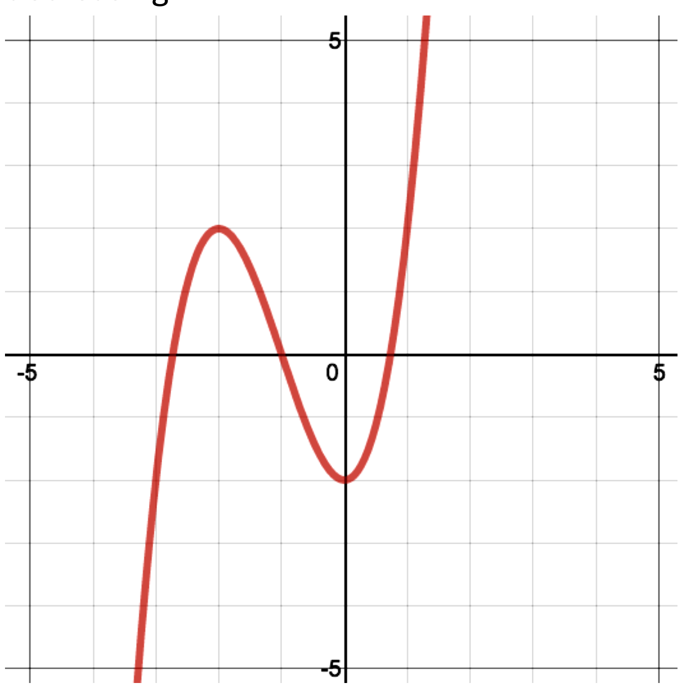
- The function is decreasing on the interval(s):

- Since the polynomial has 2 turning points, the smallest degree it can be is:

Example 2: Estimate the coordinates of each turning point and state the local maximum and minimum value of the function. Then determine the least degree the function can have.

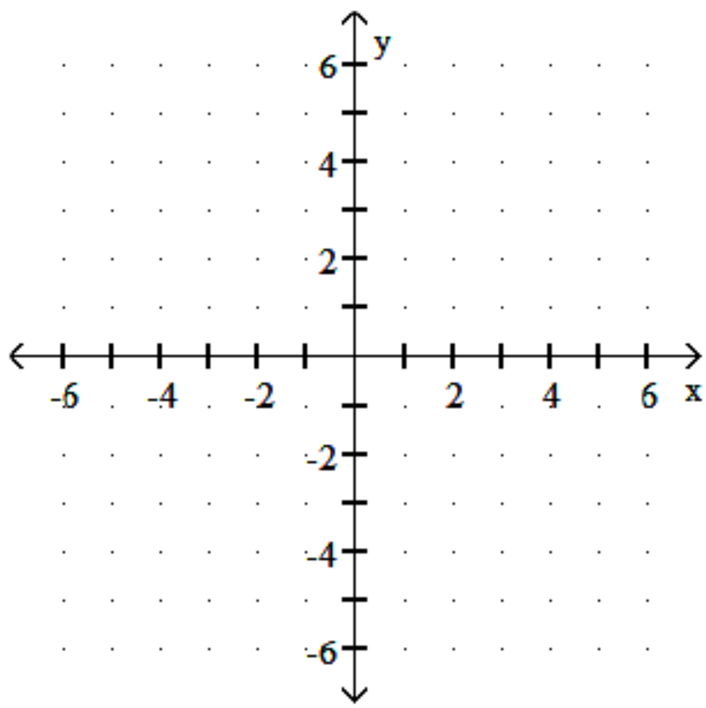


Example 3: Estimate the roots of the function, and state the intervals on which it is concave up and concave down, as well as where it is increasing and decreasing.



Example 4: Sketch a polynomial graph that has the following properties:

- It should have no real roots.
- It should have local minima at (2, 5) and (-3, 4).
- It should be decreasing from $(-\infty, -3)$.



Algebra 2

5.3 Connecting Polynomial Equations & Graphs

Warm up:

Classify of $f(x) = -8x^3 + 6x^2 + 5$ by degree and number of terms.

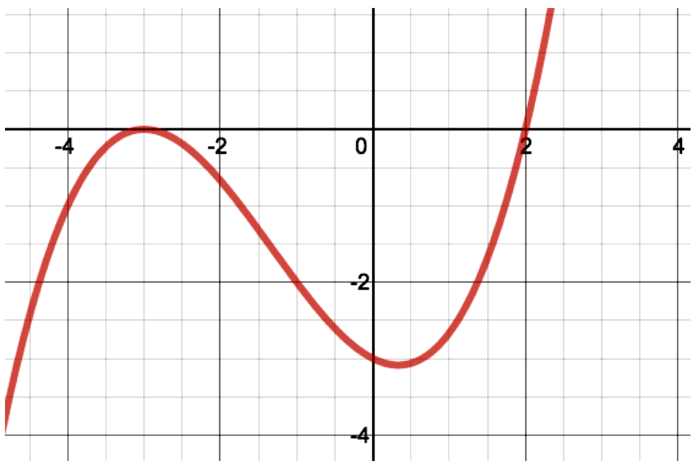
Let $f(x)$ be a polynomial function. Then the following statements are equivalent:

- $(x - k)$ is a factor of the polynomial $f(x)$
- k is a zero (root) of the polynomial $f(x)$
- If $f(k) = 0$, k is an x-intercept of the graph of the polynomial

<u>**Big Idea**</u> **Multiplicity of roots**	The **multiplicity** of a root is how many times a particular number is a zero for a given polynomial. Example: $f(x) = (x-2)^3(x-3)$ Two is a zero with multiplicity 3 Three is a zero with multiplicity 1 The function has two distinct roots, but 4 counting multiplicity

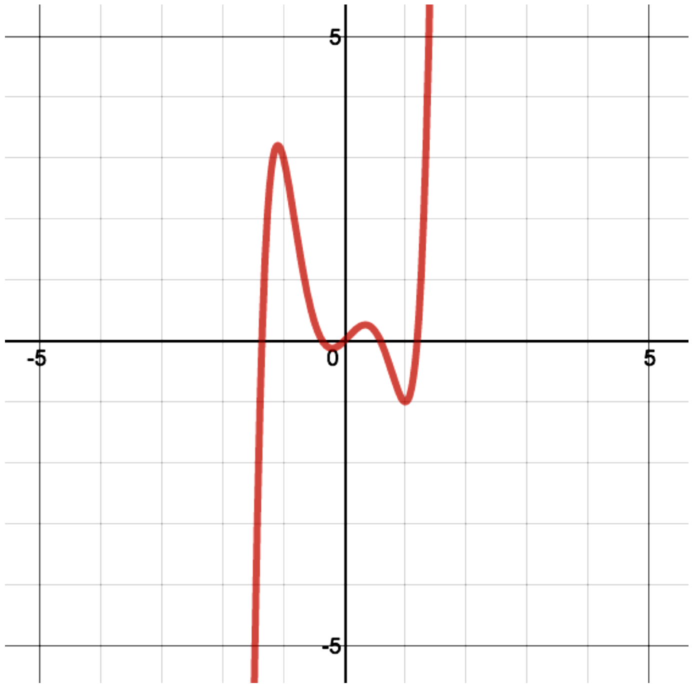
Example 1: Write a polynomial function of least possible degree that has zeros of 1, -2, and 3 (multiplicity of two).

Example 2: Estimate the real zeros (if any) of each function. Then write the equation of a cubic function that would have the same roots.



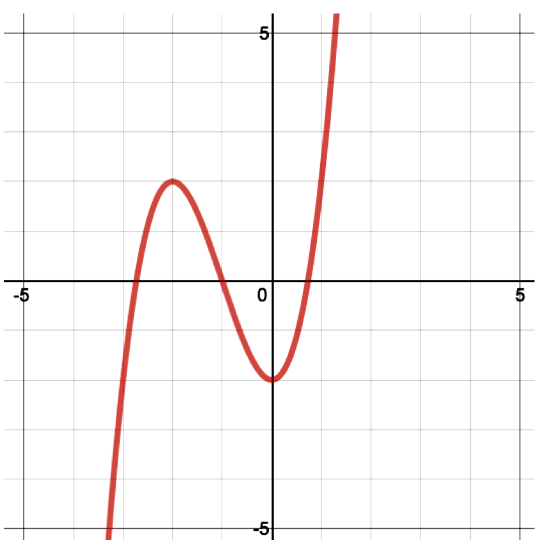
Big Idea	
Fundamental Theorem of Algebra	Any polynomial of degree n has n roots.

Example 3: Use the graph of the function $f(x) = x^7 + x^5 - 5x^3 + x^2 + x$ to approximate the real solutions to the equation $x^7 + x^5 - 5x^3 + x^2 + x = 0$. How many complex solutions does the equation have?



TRY IT: Try the problem below on your own. Then scroll down to check the answer. Ask Ms. McNabb if you need help.

Estimate the real zeros of the polynomial function shown.



ANSWER:

$x = -2.75, -1, 0.75$

5.4 DIVIDING POLYNOMIALS

Warm up:

Factor $x^2 + 5x + 6$.

Big Idea Synthetic Division	You can use **synthetic division** to divide a polynomial by a factor like $1x - k$ or $1x + k$.

Example 1: Divide the polynomial using synthetic division.

$(2x^3 + 9x^2 + 5) \div (x - 3)$

Example 2: Which of the following is possible using synthetic division? Which must be done using long division?

a. $(x^3 + 9x^2 + 5) \div (2x + 1)$

b. $(7x^3 + 9x^2 + x + 5) \div (x - 1)$

Big Idea	
Factor	Expressions that are multiplied together to equal the original expression.
Zero	A value of a variable that makes a polynomial equal to zero.
Factor Theorem	$f(k) = 0$ if and only if $x - k$ is a factor of $f(x)$
Multiplicity of a Zero	The multiplicity of a zero is the number of times the associated factor appears in the polynomial.

Example 3: Given a polynomial function $f(x)$ one of its factors, factor $f(x)$ completely.

a. $x - 3$ is a factor of $f(x) = 2x^3 - 11x^2 + 3x + 36$

b. $x + 4$ is a factor of $f(x) = x^3 + 7x^2 + 8x - 16$

Example 4: Use your work above to find the zeros of $f(x) = x^3 + 7x^2 + 8x - 16$. State the multiplicity of each zero.

Example 5: Given 2 is a zero of $f(x) = x^3 - 2x^2 + 25x - 50$, find the x-intercepts of its graph.

Big Idea	
Conjugate Zeroes Theorem	If $f(x)$ is a polynomial with real coefficients and $a + bi$ is a zero of $f(x)$, then $a - bi$ is also a zero of $f(x)$.
Fundamental Theorem of Algebra	Any polynomial of degree **n** has **n** zeroes (including both real and complex zeros).

Example 6: Given $f(x)$ is a degree 5 polynomial, and has zeros of $4i, 5 + 2i,$ and 5, find the remaining zeros of $f(x)$.

TRY IT: Try the problem below on your own. Then scroll down to check the answer. Ask Ms. McNabb if you need help.

Given that $x + 2$ is a factor of $2x^3 + 3x^2 - 5x - 6$, find all zeros of the polynomial.

ANSWER:

$x = -2, \frac{3}{2}, -1$

5.5 GRAPHING POLYNOMIALS

Warm up:

Summarize the end behavior of functions:

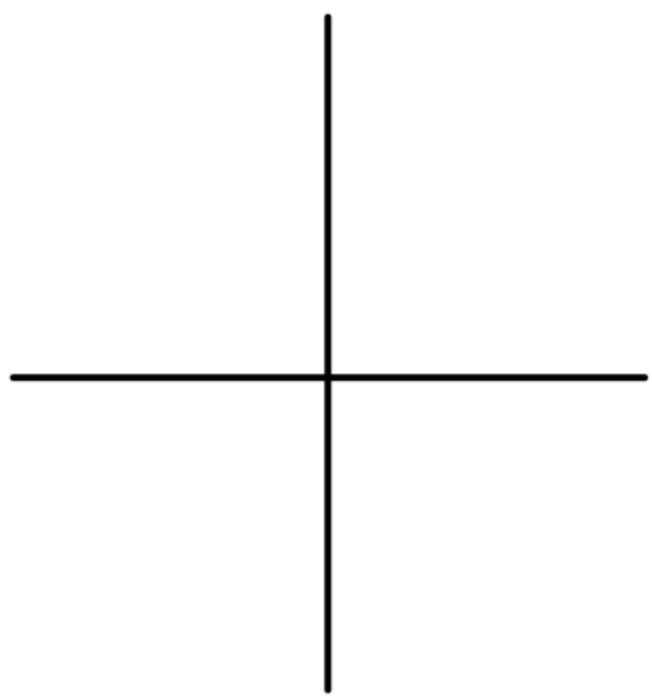
a. odd degree, positive leading coefficient

b. even degree, positive leading coefficient

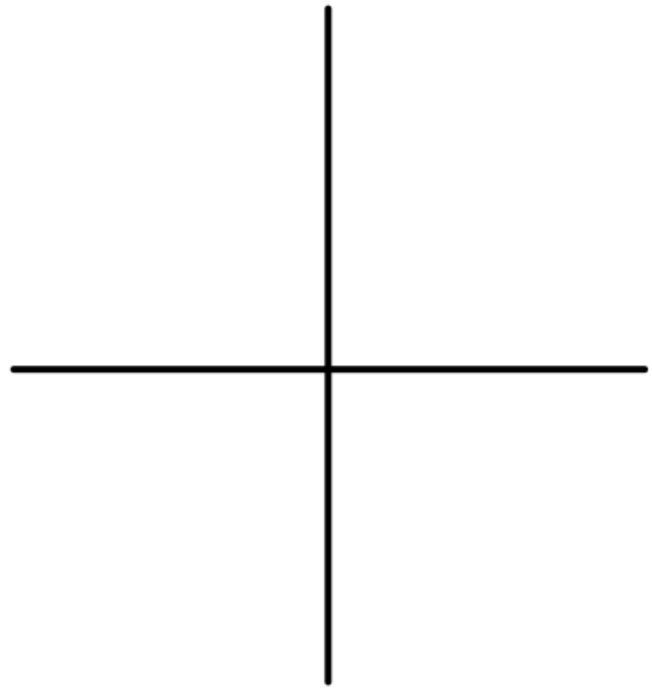
Big Idea **Graphing a Polynomial Function**	• Graph the x-intercepts (zeros) • Plot several points between each x-intercept • Draw in the correct end behavior

Example 1: Sketch a rough graph of each polynomial.

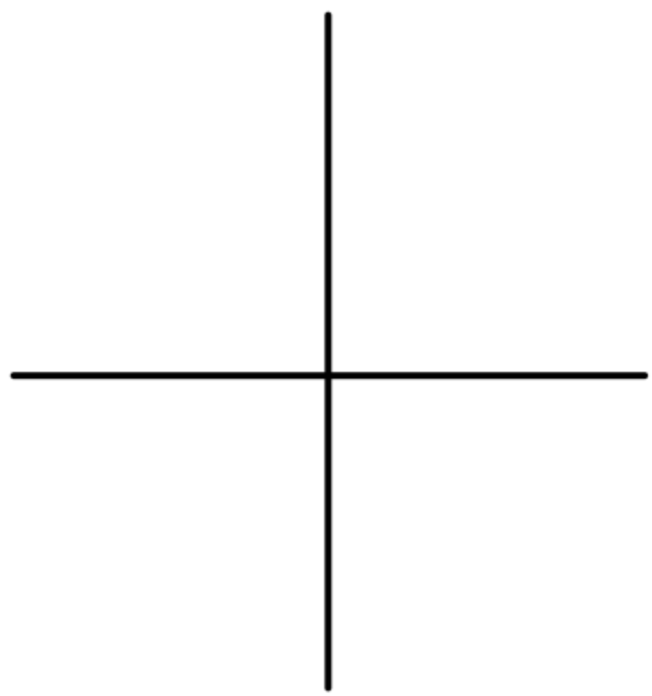
a. $f(x) = 2x(x+3)(x-2)$



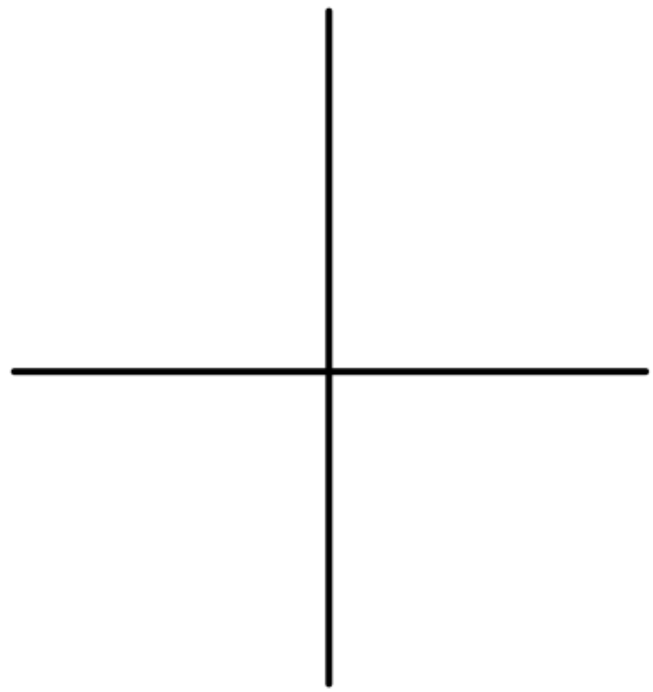
b. $4(x-2)^2(x+4)=y$



c. $g(x)=-2(x+2)^4$



d. $g(x) = x^3 - 3x^2 - 4x + 12$ (given that $x - 3$ is a factor)



TRY IT: Try the problem below on your own. Then scroll down to check the answer. Ask Ms. McNabb if you need help.

Find the x-intercepts of $y = x^2 - 4x + 4$. Hint: factor 1st!

ANSWER:

$x = 2$

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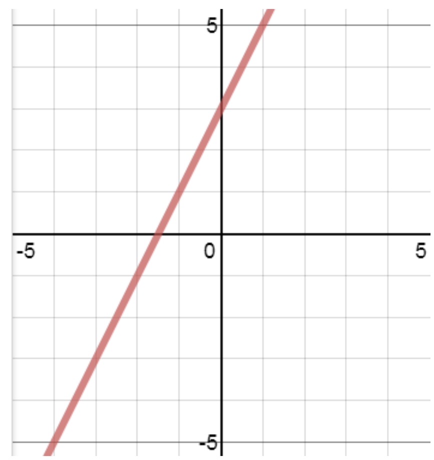
5.6 EQUATION SOLVING USING GRAPHS

Warm up:

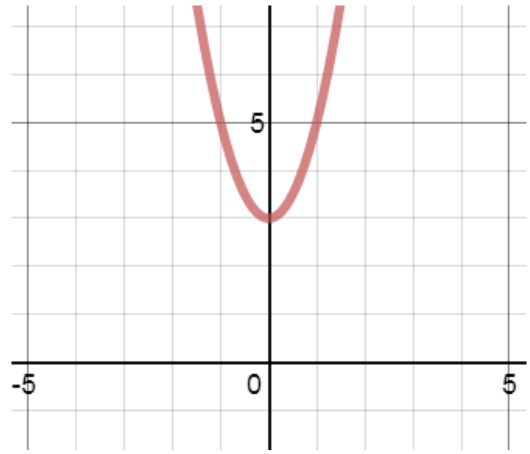
What is the maximum number of turning points that $f(x) = x^4 - 3x^2$ can have?

Example 1: Use the graph to solve the equation.

a. The graph of $f(x) = 2x + 3$ is shown.
Solve $2x + 3 = 5$.



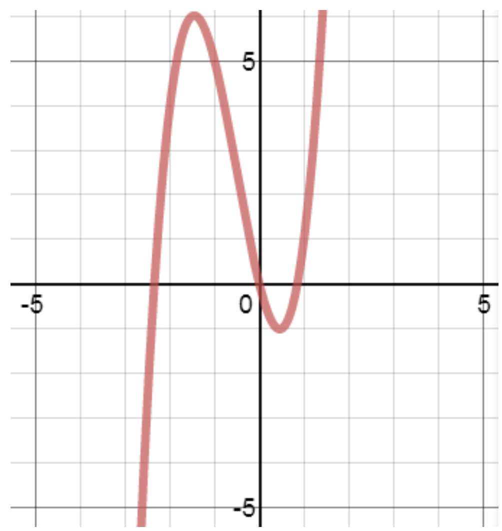
b. The graph of $f(x) = 2x^2 + 3$ is shown.



i. Solve $2x^2 + 3 = 3$.

ii. What does the graph indicate about solutions to the equation $2x^2 + 3 = 0$?

c. The graph of $f(x) = 2x^3 + 3x^2 - 4x$ is shown.
Estimate the solutions to $2x^3 + 3x^2 - 4x = -1$.



Example 2: Shown is the graph of

$$y = -241x^7 + 1060x^6 - 1870x^5 + 1650x^4 - 737x^3 + 144x^2 - 2.43x \ ,$$

a function that models one cycle of a swimmer's backstroke. The swimmer's speed in meters per second is y and x is the number of seconds since the start.

a. At what time is the swimmer going the fastest?

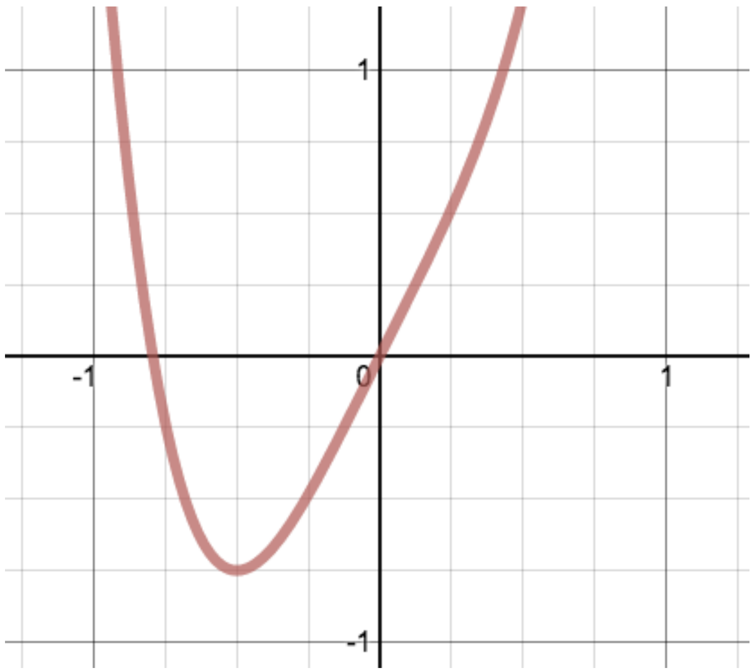
b. What is the fastest speed that the swimmer reaches?

c. How long does it take the swimmer to finish one cycle of their stroke?

d. Give approximate solutions to the equation $2 = -241x^7 + 1060x^6 - 1870x^5 + 1650x^4 - 737x^3 + 144x^2 - 2.43x$. What do the solutions to this equation tell you?

TRY IT: Try the problem below on your own. Then scroll down to check the answer. Ask Ms. McNabb if you need help.

The graph of $y = 4x^4 + 2x$ is shown.
Use the graph to find all real-valued solutions to the equation $4x^4 + 2x = -0.75$.



ANSWER:

$x = -0.5$